แบบฝึกหัดที่ 3.1

ข้อ1 หน้า83

$y = 6x^2 + 3x - 5$

$\frac{dy}{dx} = \frac{6dx^2}{dx} + \frac{3dx}{dx} - \frac{d5}{dx}$

$= 12x + 3$ #

2/83 $y = (x^2+6x)(x+4)$

$y = x^3 + 6x^2 + 4x^2 + 24x$

$y = x^3 + 10x^2 + 24x$

$\frac{dy}{dx} = \frac{dx^3}{dx} + \frac{10dx^2}{dx} + \frac{24dx}{dx}$

$= 3x^2 + 20x + 24$ #

3/83 $y = \frac{x+3}{x+4}$

$\frac{dy}{dx} = \frac{d}{dx}\frac{(x+3)}{(x+4)}$

$= \frac{(x+4)\frac{d}{dx}(x+3) - (x+3)\frac{d}{dx}(x+4)}{(x+4)^2}$

$= \frac{(x+4)(1) - (x+3)(1)}{(x+4)^2}$

$= \frac{(x+4) - (x+3)}{(x+4)^2}$

$= \frac{x+4-x-3}{(x+4)^2}$

$= \frac{1}{(x+4)^2}$ #

4/84 $y = u^2 + 3u$ | $u = 2x + 1$

$\frac{dy}{du} = 2u + 3$ | $\frac{du}{dx} = 2$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

$= 2(2u+3)$ #

5/84

$y = u^3 + 4$ | $u = 2v^2 + 1$ | $v = x + 5$

$\frac{dy}{du} = 3u^2$ | $\frac{du}{dv} = 4v$ | $\frac{dv}{dx} = 1$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dv}\frac{dv}{dx}$

$= (3u^2)(4v)(1)$

$= 12u^2v$ #

6/ $x = 3t + 2$

$\frac{dx}{dt} = 3$

$\frac{dt}{dx} = \frac{1}{3}$

$y = \sqrt{t}$

$\frac{dy}{dt} = \frac{dt^{\frac{1}{2}}}{dt}$

$= \frac{1}{2}t^{-\frac{1}{2}}$

$= \frac{1}{2t^{\frac{1}{2}}}$

$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx}$

$= \left(\frac{1}{2t^{\frac{1}{2}}}\right)\left(\frac{1}{3}\right) = \frac{1}{6\sqrt{t}}$ #

7/84 $x = t^3$ | $y = t^2$

$\frac{dx}{dt} = 3t^2$ | $\frac{dy}{dt} = 2t$

$\frac{dt}{dx} = \frac{1}{3t^2}$

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx}$$

$$= (2t)\left(\frac{1}{3t^2}\right)$$

$$= \frac{2}{3t} \quad \#$$

2/107 $y = \sin 2x + 3\cos 2x$

$$\frac{dy}{dx} = \frac{d\sin 2x}{dx} + \frac{3\,d\cos 2x}{dx}$$

$$= \cos 2x\frac{d2x}{dx} + 3\left(-\sin 2x\frac{d2x}{dx}\right)$$

$$= 2\cos 2x + 3(-\sin 2x \times 2)$$

$$= 2\cos 2x - 6\sin 2x \quad \#$$

<u>EX. 4.1</u>

1/107 $y = 3\sin(2x+1)$

$$\frac{dy}{dx} = \frac{3d\sin(2x+1)}{dx}$$

$$= 3\cos(2x+1)\frac{d(2x+1)}{dx}$$

$$= 3\cos(2x+1)(2)$$

$$= 6\cos(2x+1) \quad \#$$

3/107 $y = 4\tan 5x \cot\frac{x}{8}$

$$\frac{dy}{dx} = 4\frac{d}{dx}\tan 5x \cot\frac{x}{8}$$

$$= 4\left(\tan 5x\frac{d}{dx}\cot\frac{x}{8} + \cot\frac{x}{8}\frac{d}{dx}\tan 5x\right)$$

$$= 4\left(\tan 5x\left(-\operatorname{cosec}^2\frac{x}{8}\frac{d}{dx}\frac{x}{8}\right) + \cot\frac{x}{8}\sec^2 5x\frac{d5x}{dx}\right)$$

$$= 4\left(-\tan 5x\operatorname{cosec}^2\frac{x}{8}\frac{1}{8} + \cot\frac{x}{8}\sec^2 5x.5\right)$$

$$= 4\left(-\frac{1}{8}\tan 5x\operatorname{cosec}^2\frac{x}{8} + 5\cot\frac{x}{8}\sec^2 5x\right)$$

$\#$

$\frac{4}{107}$ $$y = \frac{\cos x + \sin x}{x}$$

$$\frac{dy}{dx} = \frac{d}{dx}\frac{(\cos x + \sin x)}{x}$$

$$= \frac{x\frac{d}{dx}(\cos x + \sin x) - (\cos x + \sin x)\frac{dx}{dx}}{x^2}$$

$$= \frac{x(-\sin x + \cos x) - (\cos x + \sin x)1}{x^2}$$

$$= \frac{-x\sin x + x\cos x - \cos x - \sin x}{x^2} \quad \#$$

$\frac{5}{107}$ $$y = x^2 + 4x - 5\sin x + 6\cot x$$

$$\frac{dy}{dx} = \frac{dx^2}{dx} + \frac{4dx}{dx} - \frac{5d\sin x}{dx} + 6\frac{d\cot x}{dx}$$

$$= 2x + 4 - 5\cos x + 6(-\csc^2 x)$$

$$= 2x + 4 - 5\cos x - 6\csc^2 x \quad \#$$

ชุด ๔.๒

$\frac{1}{121}$ $$y = \arcsin(2x+5)$$

$$\frac{dy}{dx} = \frac{d\sin^{-1}(2x+5)}{dx}$$

$$= \frac{1}{\sqrt{1-(2x+5)^2}}\frac{d(2x+5)}{dx}$$

$$= \frac{2}{\sqrt{1-(4x^2+20x+25)}}$$

$$= \frac{2}{\sqrt{1-4x^2-20x-25}}$$

$$= \frac{2}{\sqrt{-4x^2-20x-24}} \quad \#$$

$$= \frac{2}{\sqrt{4(-x^2-5x-6)}}$$

$$= \frac{2}{2\sqrt{-x^2-5x-6}} = \frac{1}{\sqrt{-x^2-5x-6}} \quad \#$$

$\frac{2}{121}$ $y = \arccos x^2$

$$\frac{dy}{dx} = \frac{d}{dx}\cos^{-1} x^2$$

$$= \frac{-1}{\sqrt{1-(x^2)^2}}\frac{dx^2}{dx}$$

$$= \frac{-2x}{\sqrt{1-x^4}} \quad \#$$

$\frac{3}{121}$ $y = \arctan 3x$

$$\frac{dy}{dx} = \frac{d}{dx}\tan^{-1} 3x$$

$$= \frac{1}{\left[1+(3x)^2\right]}\frac{d3x}{dx}$$

$$= \frac{3}{1+9x^2} \quad \#$$

<u>บฝ. 4.3</u>

$\frac{1}{141}$ $y = \ln(x^2+2x-1)$

$$\frac{dy}{dx} = \frac{d}{dx}\ln(x^2+2x-1)$$

$$= \frac{1}{(x^2+2x-1)}\frac{d(x^2+2x-1)}{dx}$$

$$= \frac{2x+2}{x^2+2x-1} \quad \#$$

$\frac{2}{141}$ $y = \log_6 (x^2+2x)(3x+4)$

$$y = \log_6(x^2+2x) + \log_6(3x+4)$$

$$\frac{dy}{dx} = \frac{d}{dx}\log_6(x^2+2x) + \frac{d}{dx}\log_6(3x+4)$$

$$= \frac{1}{(x^2+2x)}\frac{\log_6 e \, d(x^2+2x)}{dx} +$$

$$+ \frac{1}{(3x+4)}\frac{\log_6 e \, d(3x+4)}{dx}$$

$$= \frac{(2x+2)\log_6 e}{(x^2+2x)} + \frac{3\log_6 e}{(3x+4)}$$

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3/141 $y = 3^{3x^2+9x}$

$$\frac{dy}{dx} = \frac{d\,3^{3x^2+9x}}{dx}$$

$$= 3^{3x^2+9x} \ln 3 \frac{d(3x^2+9x)}{dx}$$

$$= 3^{3x^2+9x} (\ln 3)(6x+9) \#$$

4/141 $y = e^{\sin(3x+4)}$

$$\frac{dy}{dx} = \frac{d\,e^{\sin(3x+4)}}{dx}$$

$$= e^{\sin(3x+4)} \frac{d \sin(3x+4)}{dx}$$

$$= e^{\sin(3x+4)} \cos(3x+4) \frac{d(3x+4)}{dx}$$

$$= 3e^{\sin(3x+4)} \cos(3x+4) \#$$

5/142 $y = x^2 e^{\sin(2x+1)}$

$$\frac{dy}{dx} = \frac{d\,x^2 e^{\sin(2x+1)}}{dx}$$

$$= x^2 \frac{d e^{\sin(2x+1)}}{dx} + e^{\sin(2x+1)} \frac{dx^2}{dx}$$

$$= x^2 e^{\sin(2x+1)} \frac{d \sin(2x+1)}{dx} + e^{\sin(2x+1)}(2x)$$

$$= x^2 e^{\sin(2x+1)} \cos(2x+1) \frac{d(2x+1)}{dx} + e^{\sin(2x+1)}(2x)$$

$$= x^2 e^{\sin(2x+1)} \cos(2x+1)(2) + e^{\sin(2x+1)}(2x)$$

$$= 2x^2 e^{\sin(2x+1)} \cos(2x+1) + 2x e^{\sin(2x+1)} \#$$

(6)

6/142 $y = \cos(8+x^2)$

$$\frac{dy}{dx} = \frac{d\cos(8+x^2)}{dx}$$

$$= -\sin(8+x^2)\frac{d}{dx}(8+x^2)$$

$$= -\sin(8+x^2)(2x)$$

$$= -2x\sin(8+x^2) \quad \#$$

7/142 $y = e^{\cos x^2} + e^{-x^2} - e^{\sqrt{x}}$

$$\frac{dy}{dx} = \frac{de^{\cos x^2}}{dx} + \frac{de^{-x^2}}{dx} - \frac{de^{\sqrt{x}}}{dx}$$

$$= e^{\cos x^2}\frac{d\cos x^2}{dx} + e^{-x^2}\frac{d(-x^2)}{dx} - e^{\sqrt{x}}\frac{d\sqrt{x}}{dx}$$

$$= e^{\cos x^2}\left(-\sin x^2\frac{dx^2}{dx}\right) + e^{-x^2}(-2x) - e^{\sqrt{x}}\frac{dx^{\frac{1}{2}}}{dx}$$

$$= -e^{\cos x^2}\sin x^2(2x) - 2xe^{-x^2} - e^{\sqrt{x}}\left(\frac{1x^{-\frac{1}{2}}}{2}\right)$$

$$= -2xe^{\cos x^2}\sin x^2 - 2xe^{-x^2} - \frac{e^{\sqrt{x}}}{2\sqrt{x}} \quad \#$$

8/142 $y = \ln \cos 2x$

$$\frac{dy}{dx} = \frac{d}{dx} \ln \cos 2x$$

$$= \frac{1}{\cos 2x} \frac{d \cos 2x}{dx}$$

$$= \frac{-\sin 2x}{\cos 2x} \frac{d\, 2x}{dx}$$

$$= -\frac{2 \sin 2x}{\cos 2x} \quad \#$$

9/142 $y = (\log x) \cos x$

$$\frac{dy}{dx} = \frac{d}{dx} (\log x) \cos x$$

$$= \log x \frac{d}{dx} \cos x + \cos x \frac{d}{dx} \log x$$

$$= \log x (-\sin x) + \cos x \left(\frac{1}{x} \log e \right)$$

$$= -(\log x)(\sin x) + \frac{(\cos x) \log e}{x}$$

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10 $y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$.

$$\frac{dy}{dx} = \frac{d (\sin x)^{\cos x}}{dx} + \frac{d (\cos x)^{\sin x}}{dx}$$

$$= \cos x (\sin x)^{\cos x - 1} \frac{d}{dx} \sin x + (\sin x)^{\cos x} \ln (\sin x) \frac{d \cos x}{dx}$$

$$+ \sin x (\cos x)^{\sin x - 1} \frac{d \cos x}{dx} + (\cos x)^{\sin x} \ln (\cos x) \frac{d \sin x}{dx}$$

$$= \cos x (\sin x)^{\cos x - 1} \cos x - (\sin x)^{\cos x} \ln (\sin x) \sin x$$

$$- \sin x (\cos x)^{\sin x - 1} \sin x + (\cos x)^{\sin x} \ln (\cos x) \cos x$$

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ບຝ. 6.1

1/230 $\int (2x+3)\,dx$

$= 2\int x\,dx + 3\int dx$

$= \frac{2x^2}{2} + 3x + c$

$= x^2 + 3x + c$ #

2/230 $\int (x^2+3x-5)\,dx$

$= \int x^2 dx + 3\int x\,dx - 5\int dx$

$= \frac{x^3}{3} + \frac{3x^2}{2} - 5x + c$ #

3/230 $\int (3s+4)^2 ds = \int (9s^2+24s+16)\,ds$

$= 9\int s^2 ds + 24\int s\,ds + 16\int ds$

$= \frac{9s^3}{3} + \frac{24s^2}{2} + 16s + c$

$= 3s^3 + 12s^2 + 16s + c$ #

4/230 $\int (x^3+2)^2 3x^2 dx$

$= \int (x^3+2)^2 3x^2 \frac{d(x^3+2)}{3x^2}$

$= \int (x^3+2)^2 d(x^3+2)$

$= \frac{(x^3+2)^3}{3} + c$ #

8/231 $\int (x+2)^3 dx$

$= \int (x+2)^3 \frac{d(x+2)}{1}$

$= \int (x+2)^3 d(x+2)$

$= \frac{(x+2)^4}{4} + c$ #

5/230 $\int \frac{(1+x)^2\,dx}{\sqrt{x}} = \int \frac{(1+2x+x^2)\,dx}{x^{\frac{1}{2}}}$

$= \int x^{-\frac{1}{2}}(1+2x+x^2)\,dx$

$= \int (x^{-\frac{1}{2}} + 2x^{\frac{1}{2}} + x^{\frac{3}{2}})\,dx$

$= \int x^{-\frac{1}{2}}dx + 2\int x^{\frac{1}{2}}dx + \int x^{\frac{3}{2}}dx$

$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$

$= 2x^{\frac{1}{2}} + \frac{4x^{\frac{3}{2}}}{3} + \frac{2x^{\frac{5}{2}}}{5} + c$ #

6/ $\int \sqrt{x^2 - 2x^4}\,dx = \int \sqrt{x^2(1-2x^2)}\,dx$

$= \int x\sqrt{1-2x^2}\,dx$

$= \int x(1-2x^2)^{\frac{1}{2}}dx$

$= \int x(1-2x^2)^{\frac{1}{2}} \frac{d(1-2x^2)}{-4x}$

$= -\frac{1}{4}\int (1-2x^2)^{\frac{1}{2}} d(1-2x^2)$

$= -\frac{1}{4}\frac{(1-2x^2)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c$

$= \frac{-(1-2x^2)^{\frac{3}{2}}}{6} + c$ #

7/231 $\int (x^2-x)^4(2x-1)dx = \int (x^2-x)^4(2x-1)\frac{d(x^2-x)}{(2x-1)}$

$= \int (x^2-x)^4 d(x^2-x)$

$= \frac{(x^2-x)^5}{5} + c$ #

9/231 $\int (2x-5)^2 x^3 dx = \int (4x^2-20x+25)x^3 dx$

$= \int (4x^5-20x^4+25x^3)dx$

$= 4\int x^5 dx - 20\int x^4 dx + 25\int x^3 dx$

$= \frac{4x^6}{6} - \frac{20x^5}{5} + \frac{25x^4}{4} + c$

$= \frac{2x^6}{3} - 4x^5 + \frac{25x^4}{4} + c$ #

10/231 $\int \frac{x\,dx}{x^2-1} = \int \frac{x\,d(x^2-1)}{(x^2-1)(2x)}$

$= \frac{1}{2}\int \frac{d(x^2-1)}{(x^2-1)}$

$= \frac{1}{2}\ln|x^2-1| + c$ #
