

จงศึกษาตัวอย่างและสร้างแบบฝึกหัดเลียนแบบให้ได้ 100 ข้อ ส่งสัปดาห์สุดท้าย

1. กำหนดให้ $f(x) = \frac{x^3 - 1}{x}$ จงหา $f(5) - f(2)$

วิธีทำ $f(x) = \frac{x^3 - 1}{x}$

$$f(5) = \frac{5^3 - 1}{5} = \frac{124}{5}$$

$$f(2) = \frac{2^3 - 1}{2} = \frac{7}{2}$$

$$\therefore f(5) - f(2) = \frac{124}{5} - \frac{7}{2} = \frac{213}{10}$$

ตอบ

2. กำหนดให้ $f(x) = \sqrt{x^2 - 3}$ จงหา $f(-2) \cdot f(2)$

วิธีทำ $f(x) = \sqrt{x^2 - 3}$

$$f(-2) = \sqrt{(-2)^2 - 3} = \sqrt{4 - 3} = \sqrt{1}$$

$$f(2) = \sqrt{2^2 - 3} = \sqrt{4 - 3} = \sqrt{1}$$

$$f(-2) \cdot f(2) = \sqrt{1} \cdot \sqrt{1} = \sqrt{1} \quad \text{ตอบ}$$

3. $\lim_{x \rightarrow 1} (5 - 9x) = ?$

วิธีทำ $\lim_{x \rightarrow 1} (5 - 9x) = 5 - 9(1) = -4$

$$\therefore \lim_{x \rightarrow 1} (5 - 9x) = -4 \quad \text{ตอบ}$$

4. $\lim_{x \rightarrow 3} (x^2 - x - 7) = ?$

วิธีทำ $\lim_{x \rightarrow 3} (x^2 - x - 7) = 3^2 - 3 - 7$

$$= 9 - 10$$

$$\therefore \lim_{x \rightarrow 3} (x^2 - x - 7) = -1 \quad \text{ตอบ}$$

5. $\lim_{x \rightarrow 5} [(x+5)(3x-4)] = ?$

วิธีทำ $\lim_{x \rightarrow 5} [(x+5)(3x-4)] = [(5+5)(3(5)-4)] = (10)(11)$

$$\therefore \lim_{x \rightarrow 5} [(x+5)(3x-4)] = 110 \quad \text{ตอบ}$$

$$6. \lim_{x \rightarrow 1} \frac{(3x-1)^2}{(x+1)^3} = ?$$

$$\text{วิธีทำ} \quad \lim_{x \rightarrow 1} \frac{(3x-1)^2}{(x+1)^3} = \frac{(3(1)-1)^2}{(1+1)^3} = \frac{4}{8} = \frac{1}{2}$$

ตอบ

$$7. \lim_{x \rightarrow 2} \left(\frac{x-1}{x^2-1} \right) = ?$$

$$\text{วิธีทำ} \quad \lim_{x \rightarrow 2} \left(\frac{x-1}{x^2-1} \right) = \frac{2-1}{2^2-1} = \frac{1}{3}$$

ตอบ

$$8. \lim_{x \rightarrow \infty} \frac{2x+3}{4x-5} = ?$$

$$\begin{aligned} \text{วิธีทำ} \quad \lim_{x \rightarrow \infty} \frac{2x+3}{4x-5} &= \lim_{x \rightarrow \infty} \frac{\frac{2x}{x} + \frac{3}{x}}{\frac{4x}{x} - \frac{5}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{2 + \frac{3}{x}}{4 - \frac{5}{x}} \\ &= \frac{\lim_{x \rightarrow \infty} 2 + \lim_{x \rightarrow \infty} \frac{3}{x}}{\lim_{x \rightarrow \infty} 4 - \lim_{x \rightarrow \infty} \frac{5}{x}} \\ &= \frac{2+0}{4-0} = \frac{2}{4} = \frac{1}{2} \end{aligned}$$

ตอบ

$$9. \lim_{x \rightarrow \infty} \frac{7x^2-2}{3x^2+10x-100} = ?$$

$$\begin{aligned} \text{วิธีทำ} \quad \lim_{x \rightarrow \infty} \frac{7x^2-2}{3x^2+10x-100} &= \lim_{x \rightarrow \infty} \frac{\frac{7x^2}{x^2} - \frac{2}{x^2}}{\frac{3x^2}{x^2} + \frac{10x}{x^2} - \frac{100}{x^2}} \\ &= \lim_{x \rightarrow \infty} \frac{7 - \frac{2}{x^2}}{3 + \frac{10}{x} - \frac{100}{x^2}} \\ &= \frac{7-0}{3+0-0} = \frac{7}{3} \end{aligned}$$

$$\therefore \lim_{x \rightarrow \infty} \frac{7x^2-2}{3x^2+10x-100} = \frac{7}{3}$$

ตอบ

$$10. \lim_{x \rightarrow \infty} \frac{2x^3}{x^2+1} = ?$$

$$\begin{aligned} \text{วิธีทำ} \quad \lim_{x \rightarrow \infty} \frac{2x^3}{x^2+1} &= \lim_{x \rightarrow \infty} \frac{2x^3}{\frac{x^2}{x^3} + \frac{1}{x^3}} \\ &= \lim_{x \rightarrow \infty} \frac{2}{\frac{1}{x} + \frac{1}{x^3}} \\ &= \frac{\lim_{x \rightarrow \infty} 2}{\lim_{x \rightarrow \infty} \left(\frac{1}{x} + \frac{1}{x^3} \right)} \\ &= \frac{2}{0+0} = \frac{2}{0} = \infty \quad \text{ตอบ} \end{aligned}$$

$$11. \lim_{x \rightarrow \infty} \frac{x}{x^2+5} = ?$$

$$\begin{aligned} \text{วิธีทำ} \quad \therefore \lim_{x \rightarrow \infty} \frac{x}{x^2+5} &= \lim_{x \rightarrow \infty} \frac{\frac{x}{x^2}}{\frac{x^2}{x^2} + \frac{5}{x^2}} \\ &= \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1 + \frac{5}{x^2}} = \frac{0}{1+0} \\ &= \frac{0}{1} = 0 \\ \therefore \lim_{x \rightarrow \infty} \frac{x}{x^2+5} &= 0 \quad \text{ตอบ} \end{aligned}$$

$$12. \text{ ถ้า } y = 10 \text{ จงหาค่าอนุพันธ์ของ } y$$

$$\begin{aligned} \text{วิธีทำ} \quad y &= 10 \\ \frac{dy}{dx} &= \frac{d10}{dx} = 0 \quad \text{ตอบ} \end{aligned}$$

$$13. \text{ ถ้า } y = -2x \text{ จงหาค่าอนุพันธ์ของ } y$$

$$\begin{aligned} \text{วิธีทำ} \quad y &= -2x \\ \therefore \frac{dy}{dx} &= \frac{d}{dx} -2x = -2 \frac{d}{dx} x = -2 \quad \text{ตอบ} \end{aligned}$$

$$14. \text{ ถ้า } y = x^6 \text{ จงหาค่าอนุพันธ์ของ } y$$

$$\begin{aligned} \text{วิธีทำ} \quad y &= x^6 \\ \therefore \frac{dy}{dx} &= \frac{d}{dx} x^6 = 6x^{6-1} \frac{dx}{dx} = 6x^5 \frac{dx}{dx} = 6x^5 \quad \text{ตอบ} \end{aligned}$$

15. ถ้า $y = 4x^3 - 5x^2 + 7x - 10$

วิธีทำ
$$\begin{aligned} \frac{dy}{dx} &= \frac{d(4x^3)}{dx} - \frac{d(5x^2)}{dx} + \frac{d7x}{dx} - \frac{d10}{dx} \\ &= 4 \frac{dx^3}{dx} - 5 \frac{dx^2}{dx} + 7 - 0 \\ &= 4 \left(2x^{3-1} \frac{dx}{dx} \right) - 5 \left(2x^{2-1} \frac{dx}{dx} \right) + 7 \\ \therefore \frac{dy}{dx} &= 12x^2 - 10x + 7 \end{aligned}$$

ตอบ

16. ถ้า $y = x^5 + 5x^4 - 10x^2 + 6$ จงหา y'

วิธีทำ
$$\begin{aligned} y &= x^5 + 5x^4 - 10x^2 + 6 \\ \frac{dy}{dx} &= \frac{d}{dx}(x^5) + 5 \frac{d}{dx}(x^4) - 10 \frac{d}{dx}(x^2) + \frac{d(6)}{dx} \\ &= 5x^{5-1} + 5(4)x^{4-1} - 10(2)x^{2-1} + 0 \\ &= 5x^4 + 20x^3 - 20x \end{aligned}$$

ตอบ

17. ถ้า $y = 3x^{\frac{1}{2}} - x^{\frac{3}{2}} + 2x^{\frac{-1}{2}}$ จงหา $\frac{dy}{dx}$

วิธีทำ
$$\begin{aligned} y &= 3x^{\frac{1}{2}} - x^{\frac{3}{2}} + 2x^{\frac{-1}{2}} \\ \frac{dy}{dx} &= 3 \frac{d}{dx}(x^{\frac{1}{2}}) - \frac{d}{dx}(x^{\frac{3}{2}}) + 2 \frac{d}{dx}(x^{\frac{-1}{2}}) \\ &= 3 \left(\frac{1}{2} \right) x^{\frac{1}{2}-1} - \frac{3}{2} x^{\frac{3}{2}-1} + 2 \left(\frac{-1}{2} \right) x^{\frac{-1}{2}-1} \\ &= \frac{3}{2} x^{-\frac{1}{2}} - \frac{3}{2} x^{\frac{1}{2}} - x^{-\frac{3}{2}} \\ \therefore \frac{dy}{dx} &= \frac{3}{2\sqrt{x}} + \frac{3\sqrt{x}}{2} - \frac{1}{x^{\frac{3}{2}}} \end{aligned}$$

ตอบ

18. ถ้า $y = \sqrt{2x} + 2\sqrt{x}$ จงหา y'

วิธีทำ
$$\begin{aligned} y &= \sqrt{2x} + 2\sqrt{x} \\ y &= (2x)^{\frac{1}{2}} + 2x^{\frac{1}{2}} \\ \frac{dy}{dx} &= \frac{d}{dx}(2x)^{\frac{1}{2}} + 2 \frac{d}{dx}(x^{\frac{1}{2}}) \\ &= \frac{1}{2} (2x)^{\frac{1}{2}-1} \frac{d}{dx}(2x) + 2 \left(\frac{1}{2} \right) x^{\frac{1}{2}-1} \\ &= \frac{1}{2} (2x)^{-\frac{1}{2}} 2 \frac{dx}{dx} + x^{-\frac{1}{2}} \\ &= (2x)^{-\frac{1}{2}} (1) + x^{-\frac{1}{2}} = \frac{1}{\sqrt{2x}} + \frac{1}{\sqrt{x}} \\ \therefore \frac{dy}{dx} &= \frac{1+\sqrt{2}}{\sqrt{2x}} \end{aligned}$$

ตอบ

19. ถ้า $y = (1-5x)^6$ จงหา y'

วิธีทำ $y = (1-5x)^6$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(1-5x)^6 \\ &= 6(1-5x)^5 \frac{d}{dx}(1-5x) \\ &= 6(1-5x)^5 (0-5(1)) \\ &= 6(-5)(1-5x)^5\end{aligned}$$

$$\therefore \frac{dy}{dx} = -30(1-5x)^5$$

ตอบ

20. ถ้า $y = (4x)(x+1)$ จงหา y'

วิธีทำ $y = (4x)(x+1)$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(4x)(x+1) \\ &= 4x \frac{d}{dx}(x+1) + (x+1) \frac{d}{dx}4x \\ &= 4x(1) + (x+1)4 \\ &= 4x + 4x + 4 \\ &= 8x + 4\end{aligned}$$

ตอบ

21. ถ้า $y = (x^2-1)(3x)$ จงหา y'

วิธีทำ $y = (x^2-1)(3x)$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(x^2-1)(3x) \\ &= (x^2-1) \frac{d}{dx}(3x) + (3x) \frac{d}{dx}(x^2-1) \\ &= (x^2-1)(3) + (3x)(2x) \\ &= 3x^2 - 3 + 6x^2 = 9x^2 - 3\end{aligned}$$

ตอบ

22. ถ้า $y = (5+6x)(3x-1)$ จงหา y'

วิธีทำ $y = (5+6x)(3x-1)$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(5+6x)(3x-1) \\ &= (5+6x) \frac{d}{dx}(3x-1) + (3x-1) \frac{d}{dx}(5+6x) \\ &= (5+6x)(3) + (3x-1)6 \\ &= 15+18x + 18x - 6 \\ &= 36x + 9\end{aligned}$$

ตอบ

23. ถ้า $y = (x^2 + 1)(7 - 3x)$ จงหา y'

วิธีทำ $y = (x^2 + 1)(7 - 3x)$

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (x^2 + 1)(7 - 3x) \\ &= (x^2 + 1) \frac{d}{dx} (7 - 3x) + (7 - 3x) \frac{d}{dx} (x^2 + 1) \\ &= (x^2 + 1)(-3) + (7 - 3x)(2x) \\ &= -3x^2 - 3 + 14x - 6x^2 \\ &= -9x^2 + 11 \quad \text{ตอบ} \end{aligned}$$

24. ถ้า $y = \frac{(3x - 1)}{x}$ จงหา y'

วิธีทำ $y = \frac{(3x - 1)}{x}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \frac{(3x - 1)}{x} \\ &= \frac{1}{x^2} \left[x \frac{d}{dx} (3x - 1) - (3x - 1) \frac{d}{dx} x \right] \\ &= \frac{1}{x^2} [x(3) - (3x - 1)(1)] \\ &= \frac{1}{x^2} [3x - 3x + 1] = \frac{1}{x^2} \quad \text{ตอบ} \end{aligned}$$

25. ถ้า $y = \frac{3x - 5}{x^2 + 2}$ จงหา y'

วิธีทำ $y = \frac{3x - 5}{x^2 + 2}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \frac{3x - 5}{x^2 + 2} \\ &= \frac{1}{(x^2 + 2)^2} \left[(x^2 + 2) \frac{d}{dx} (3x - 5) - (3x - 5) \frac{d}{dx} (x^2 + 2) \right] \\ &= \frac{1}{(x^2 + 2)^2} [(x^2 + 2)(3) - (3x - 5)(2x)] \\ &= \frac{1}{(x^2 + 2)^2} [3x^2 + 6 - 6x^2 + 10x] \\ &= \frac{1}{(x^2 + 2)^2} [-3x^2 + 10x + 6] \quad \text{ตอบ} \end{aligned}$$

26. ถ้า $y = (2x)^4$ จงหา y'

วิธีทำ $y = (2x)^4$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(2x)^4 \\ &= 4(2x)^{4-1} \frac{d}{dx}(2x) = 8(2x)^3 \quad \text{ตอบ}\end{aligned}$$

27. ถ้า $y = (x-8)^{-2}$ จงหา y'

วิธีทำ $y = (x-8)^{-2}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(x-8)^{-2} = -2(x-8)^{-2-1} \frac{d}{dx}(x-8) \\ &= -2(x-8)^{-3} \frac{d}{dx}(x-8) \\ &= -2(x-8)^{-3} \quad \text{ตอบ}\end{aligned}$$

28. ถ้า $y = \sqrt{3+4x-x^2}$ จงหา y'

วิธีทำ $y = \sqrt{3+4x-x^2} = (3+4x-x^2)^{\frac{1}{2}}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(3+4x-x^2)^{\frac{1}{2}} \\ &= \frac{1}{2}(3+4x-x^2)^{-\frac{1}{2}} \frac{d}{dx}(3+4x-x^2) \\ &= \frac{1}{2}(3+4x-x^2)^{-\frac{1}{2}}(4-2x) \\ &= \frac{1}{2}2(2-x)(3+4x-x^2)^{-\frac{1}{2}} \\ &= \frac{2-x}{(3+4x-x^2)^{\frac{1}{2}}} = \frac{2-x}{\sqrt{3+4x-x^2}} = \frac{2-x}{y}\end{aligned}$$

$$\therefore \frac{dy}{dx} = \frac{2-x}{y}$$

ตอบ

29. ถ้า $\theta = \frac{3r+2}{2r+3}$ จงหา $\frac{d\theta}{dr}$

วิธีทำ $\theta = \frac{3r+2}{2r+3}$

$$\begin{aligned}\frac{d\theta}{dr} &= \frac{d}{dr} \left[\frac{3r+2}{2r+3} \right] \\ &= \frac{(2r+3) \frac{d}{dr}(3r+2) - (3r+2) \frac{d}{dr}(2r+3)}{(2r+3)^2} \\ &= \frac{(2r+3) \left[3 \frac{dr}{dr} + \frac{d(2)}{dr} \right] - (3r+2) \left[2 \frac{dr}{dr} + \frac{d3}{dr} \right]}{(2r+3)^2}\end{aligned}$$

$$\begin{aligned}
&= \frac{(2r+3)(3(1)+0) - (3r+2)(2(1)+0)}{(2r+3)^2} \\
&= \frac{3(2r+3) - 2(3r+2)}{(2r+3)^2} \\
&= \frac{6r+9-6r-4}{(2r+3)^2} = \frac{5}{(2r+3)^2}
\end{aligned}$$

$$\frac{d\theta}{dr} = \frac{5}{(2r+3)^2}$$

ตอบ

30. กำหนดให้ $y = (x^2 + 4)^2(2x^3 - 1)^3$ จงหา y'

วิธีทำ $y = (x^2 + 4)^2(2x^3 - 1)^3$

$$\begin{aligned}
\therefore \frac{dy}{dx} &= \frac{d}{dx} [(x^2 + 4)^2(2x^3 - 1)^3] \\
&= (x^2 + 4)^2 \frac{d}{dx} (2x^3 - 1)^3 + (2x^3 - 1)^3 \frac{d}{dx} (x^2 + 4)^2 \\
&= (x^2 + 4)^2 3(2x^3 - 1)^2 \frac{d}{dx} (2x^3 - 1) + (2x^3 - 1)^3 2(x^2 + 4) \frac{d}{dx} (x^2 + 4) \\
&= 3(x^2 + 4)^2 (2x^3 - 1)^2 \left[2 \frac{d(x^3)}{dx} - \frac{d(1)}{dx} \right] + (2x^3 - 1)^3 2(x^2 + 4) \left[\frac{d(x^2)}{dx} + \frac{d(4)}{dx} \right] \\
&= 3(x^2 + 4)^2 (2x^3 - 1)^2 (6x^2 - 0) + 2(2x^3 - 1)^3 (x^2 + 4)(2x + 0) \\
&= 18x^2(x^2 + 4)^2(2x^3 - 1)^2 + 4x(2x^3 - 1)^3(x^2 + 4) \\
&= 2x(x^2 + 4)(2x^3 - 1)^2 \{ 9x(x^2 + 4) + 2(2x^3 - 1) \} \\
&= 2x(x^2 + 4)(2x^3 - 1)^2 (9x^3 + 36x + 4x^3 - 2) \\
\therefore y' &= 2x(x^2 + 4)(2x^3 - 1)^2 (13x^3 + 36x - 2)
\end{aligned}$$

ตอบ

31. กำหนดให้ $f(x) = x^3 - 4x + 1$ จงหา $f'(-1)$

วิธีทำ $f(x) = x^3 - 4x + 1$

$$\begin{aligned}
f'(x) &= \frac{d}{dx} (x^3 - 4x + 1) \\
&= \frac{d}{dx} (x^3) 4 - \frac{dx}{dx} + \frac{d(1)}{dx}
\end{aligned}$$

$$\therefore f'(x) = 3x^2 - 4$$

แทนค่า $x = -1$ ลงใน $f'(x)$ จะได้ $f'(-1) = 3(-1)^2 - 4 = 3 - 4 = -1$

ตอบ

32. กำหนดให้ $y = \left[\frac{x}{1+x} \right]^5$ จงหา y'

วิธีทำ $y = \left[\frac{x}{1+x} \right]^5$

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx} \left[\frac{x}{1+x} \right]^5 \\
&= 5 \left[\frac{x}{1+x} \right]^4 \frac{d}{dx} \left[\frac{x}{1+x} \right]
\end{aligned}$$

$$\begin{aligned}
&= 5 \left[\frac{x}{1+x} \right]^4 \left\{ \frac{(1+x) \frac{dx}{dx} - x \frac{d}{dx} (1+x)}{(1+x)^2} \right\} \\
&= \frac{5x^4}{(1+x)^4} \left\{ \frac{(1+x)(1) - x \left[\frac{d(1)}{dx} + \frac{dx}{dx} \right]}{(1+x)^2} \right\} \\
&= \frac{5x^4}{(1+x)^4} \left\{ \frac{(1+x) - x(0+1)}{(1+x)^2} \right\} \\
&= \frac{5x^4}{(1+x)^4} \left\{ \frac{1+x-x}{(1+x)^2} \right\} \\
\therefore \frac{dy}{dx} &= \frac{5x^4}{(1+x)^6}
\end{aligned}$$

ตอบ

33. ถ้า $y = (x^2 + 3)^4 (2x^3 - 5)^3$ จงหา y'

วิธีทำ

$$y = (x^2 + 3)^4 (2x^3 - 5)^3$$

$$y' = \frac{d}{dx} (x^2 + 3)^4 (2x^3 - 5)^3$$

$$= (x^2 + 3)^4 \frac{d}{dx} (2x^3 - 5)^3 + (2x^3 - 5)^3 \frac{d}{dx} (x^2 + 3)^4$$

$$= (x^2 + 3)^4 3(2x^3 - 5)^2 6x^2 + (2x^3 - 5)^3 4(x^2 + 3)^3 2x$$

$$= 18x^2 (x^2 + 3)^4 (2x^3 - 5)^2 + 8x (2x^3 - 5)^3 (x^2 + 3)^3$$

$$= 2x (x^2 + 3)^3 (2x^3 - 5)^2 \{ 9x(x^2 + 3) + 4(2x^3 - 5) \}$$

$$= 2x (x^2 + 3)^3 (2x^3 - 5)^2 \{ 18x^3 + 27x + 8x^3 - 20 \}$$

$$= 2x (x^2 + 3)^3 (2x^3 - 5)^2 (26x^3 + 27x - 20)$$

ตอบ

34. ถ้า $f(x) = (3x - x^3 + 1)^4$ จงหา $f'(x)$

วิธีทำ

$$f(x) = (3x - x^3 + 1)^4$$

$$f'(x) = \frac{d}{dx} (3x - x^3 + 1)^4$$

$$= 4(3x - x^3 + 1)^3 (3 - 3x^2)$$

$$= 12(1 - x^2)(3x - x^3 + 1)^3$$

ตอบ

35. ถ้า $y = \left(\frac{x^3-1}{2x^3+1}\right)^4$ จงหา y'

วิธีทำ

$$y = \left(\frac{x^3-1}{2x^3+1}\right)^4$$

$$y' = \frac{d}{dx} \left(\frac{x^3-1}{2x^3+1}\right)^4$$

$$= 4 \left[\frac{x^3-1}{2x^3+1}\right]^3 \frac{d}{dx} \left(\frac{x^3-1}{2x^3+1}\right)$$

$$= 4 \left[\frac{x^3-1}{2x^3+1}\right]^3 \frac{(2x^3+1)3x^2 - (x^3-1)6x^2}{(2x^3+1)^2}$$

$$= 4 \left[\frac{x^3-1}{2x^3+1}\right]^3 \left\{ \frac{6x^5 + 3x^2 - 6x^5 + 6x^2}{(2x^3+1)^3} \right\}$$

$$= \frac{36x^2(x^3-1)^3}{(2x^3+1)^6}$$

ตอบ

36. ถ้า $s = \frac{t^2+2}{3-t^2}$ จงหา $s'(t)$

วิธีทำ

$$s = \frac{t^2+2}{3-t^2}$$

$$\frac{ds}{dt} = \frac{d}{dt} \left(\frac{t^2+2}{3-t^2}\right)$$

$$= \frac{(3-t^2) \frac{d}{dt}(t^2+2) - (t^2+2) \frac{d}{dt}(3-t^2)}{(3-t^2)^2}$$

$$= \frac{2t(3-t^2) + 2t(t^2+2)}{(3-t^2)^2}$$

$$= \frac{6t - 2t^3 + 2t^3 + 4t}{(3-t^2)^2}$$

$$= \frac{10t}{(3-t^2)^2}$$

ตอบ

37. ถ้า $y = u^2 + 4$ และ $u^2 = x^2 + 2x$ จงหา $\frac{dy}{dx}$

วิธีทำ

$$y = u^2 + 4$$

$$\frac{dy}{du} = \frac{d}{du}(u^2 + 4)$$

$$= \frac{d}{du}(u^2) + \frac{d}{du}(4)$$

$$\therefore \frac{dy}{du} = 2u \quad \#$$

$$u = x^2 + 2x$$

$$\begin{aligned}\frac{du}{dx} &= \frac{d}{dx}(x^2 + 2x) \\ &= \frac{d}{dx}(x^2) + 2\frac{dx}{dx}\end{aligned}$$

$$\therefore \frac{du}{dx} = 2x + 2 \quad \#$$

โดย chain Rule จะได้ $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$$= (2u)(2x + 2) = 2(x^2 + 2x)(2x + 2)$$

$$\therefore \frac{dy}{dx} = 4x(x + 2)(x + 1)$$

ตอบ

38. กำหนดให้ $y = 4 \tan 5x$ จงหา y'

วิธีทำ $y = 4 \tan 5x$

$$\begin{aligned}\frac{dy}{dx} &= 4 \frac{d}{dx}(\tan 5x) \\ &= 4 \sec^2 5x \frac{d}{dx}(5x) \\ &= 4 \sec^2 5x \cdot 5 \frac{dx}{dx} \\ &= 20 \sec^2 5x(1)\end{aligned}$$

$$\therefore y' = \frac{dy}{dx} = 20 \sec^2 5x$$

ตอบ

39. กำหนดให้ $y = \frac{1}{4} \csc 4x$ จงหา y'

วิธีทำ $y = \frac{1}{4} \csc 4x$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(\csc 4x) \\ &= \frac{1}{4} \frac{d}{dx}(\csc 4x) \\ &= \frac{1}{4} (\csc 4x)(\cot 4x) \frac{d}{dx}(4x) \\ &= \frac{-1}{4} \csc 4x \cot 4x \cdot 4 \frac{dx}{dx}\end{aligned}$$

$$\therefore \frac{dy}{dx} = -\csc 4x \cot 4x$$

ตอบ

40. กำหนดให้ $y = \sin x - x \cos x + x^2 + 4x + 3$ จงหา $\frac{dy}{dx}$

วิธีทำ $y = \sin x - x \cos x + x^2 + 4x + 3$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x) - \frac{d}{dx}(x \cos x) + \frac{d}{dx}(x^2) + \frac{d}{dx}(4x) + \frac{d}{dx}(3)$$

$$\begin{aligned}
&= \cos x \frac{dx}{dx} - \left\{ x \frac{d}{dx}(\cos x) + \cos x \frac{dx}{dx} \right\} + 2x + 4 \frac{dx}{dx} + 0 \\
&= \cos x - \left\{ x(-\sin x) \frac{dx}{dx} + \cos x(1) \right\} + 2x + 4(1) \\
&= \cos x + x \sin x - \cos x + 2x + 4
\end{aligned}$$

$$\therefore \frac{dy}{dx} = x \sin x + 2x + 4$$

ตอบ

41. กำหนดให้ $y = \sin^2(3x-2)$ จงหา $\frac{dy}{dx}$

วิธีทำ $y = \sin^2(3x-2)$

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx} [\sin(3x-2)]^2 \\
&= 2 \sin(3x-2) \frac{d}{dx} [\sin(3x-2)] \\
&= 2 \sin(3x-2) \cos(3x-2) \frac{d}{dx} (3x-2) \\
&= 2 \sin(3x-2) \cos(3x-2) \left[3 \frac{dx}{dx} - \frac{d}{dx} (2) \right] \\
&= 3(2) \sin(3x-2) \cos(3x-2) \\
&= 3 \sin 2(3x-2) \because 2 \sin A \cos A = \sin 2A
\end{aligned}$$

$$\therefore \frac{dy}{dx} = 3 \sin(6x-4)$$

ตอบ

42. กำหนดให้ $y = x^2 \sin x + 2x \cos x - 2 \sin x$ จงหา y'

วิธีทำ $y = x^2 \sin x + 2x \cos x - 2 \sin x$

$$\begin{aligned}
y' &= \frac{d}{dx} (x^2 \sin x) + 2 \frac{d}{dx} (x \cos x) - 2 \frac{d}{dx} (\sin x) \\
&= x^2 \frac{d}{dx} \sin x + \sin x \frac{d}{dx} (x^2) + 2 \left\{ x \frac{d}{dx} (\cos x) + \cos x \frac{dx}{dx} \right\} - 2 \cos x \\
&= x^2 \cos x + 2x \sin x - 2x \sin x + 2 \cos x - 2 \cos x
\end{aligned}$$

$$\therefore y' = x^2 \cos x$$

ตอบ

43. กำหนดให้ $h(x) = \sin(\cos x)$ จงหา $h'(x)$

วิธีทำ $h(x) = \sin(\cos x)$

$$\begin{aligned}
h'(x) &= \frac{d}{dx} [\sin(\cos x)] \\
&= \cos(\cos x) \frac{d}{dx} (\cos x) = \cos(\cos x) \left(-\sin x \frac{dx}{dx} \right)
\end{aligned}$$

$$\therefore h'(x) = -\sin x \cos(\cos x)$$

ตอบ

44. กำหนดให้ $y = \sin x \tan x$ จงหา y'

วิธีทำ $y = \sin x \tan x$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \sin x \tan x \\ &= \sin x \frac{d}{dx} \tan x + \tan x \frac{d}{dx} \sin x \\ &= \sin x \sec^2 x + \tan x \cos x \quad \text{ตอบ}\end{aligned}$$

45. กำหนดให้ $y = x \tan x$ จงหา y'

วิธีทำ $y = x \tan x$

$$\begin{aligned}y' &= \frac{d}{dx} (x \tan x) \\ &= x \frac{d}{dx} (\tan x) + \tan x \frac{dx}{dx} \\ &= x \sec^2 \frac{dx}{dx} + \tan x (1)\end{aligned}$$

$$\therefore y' = x \sec^2 x + \tan x$$

ตอบ

46. กำหนดให้ $y = \frac{\cos x}{x}$ จงหา y'

วิธีทำ $y = \frac{\cos x}{x}$

$$\begin{aligned}y' &= \frac{d}{dx} \left[\frac{\cos x}{x} \right] \\ &= \frac{x \frac{d}{dx} (\cos x) - \cos x \frac{dx}{dx}}{x^2} \\ &= \frac{x(-\sin x) \frac{dx}{dx} - \cos x (1)}{x^2} \\ \therefore y' &= \frac{-x \sin x - \cos x}{x^2}\end{aligned}$$

ตอบ

47. กำหนดให้ $y = \arcsin 3x$ จงหา $\frac{dy}{dx}$

วิธีทำ $y = \arcsin 3x$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [\arcsin 3x] \\ &= \frac{1}{\sqrt{1-(3x)^2}} \cdot \frac{d}{dx} (3x) \\ &= \frac{1}{\sqrt{1-9x^2}} \cdot 3 \frac{dx}{dx} \\ \therefore \frac{dy}{dx} &= \frac{3}{\sqrt{1-9x^2}}\end{aligned}$$

ตอบ

48. กำหนดให้ $f(x) = \log 4x$ จงหา $f'(x)$

วิธีทำ $f(x) = \log 4x$ จะได้

$$\begin{aligned} f'(x) &= \frac{d}{dx} [\log 4x] \\ &= \frac{1}{4x} \cdot \log e \frac{d}{dx} (4x) \\ &= \frac{4}{4x} \cdot \log e \\ \therefore f'(x) &= \frac{1}{x} \cdot \log e \end{aligned}$$

49.. กำหนดให้ $f(x) = x \cdot \log x$ จงหา $f'(x)$

วิธีทำ $f(x) = x \cdot \log x$

$$\begin{aligned} f'(x) &= \frac{d}{dx} (x \cdot \log x) \\ &= x \frac{d}{dx} (\log x) + \log x \frac{dx}{dx} \\ &= x \left(\frac{1}{x} \right) \cdot \log e \frac{dx}{dx} + \log x (1) \\ \therefore f'(x) &= \log e + \log x \end{aligned}$$

ตอบ

50. กำหนดให้ $y = e^{x^3}$ จงหา y'

วิธีทำ $y = e^{x^3}$ จะได้

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (e^{x^3}) \\ &= e^{x^3} \frac{d}{dx} (x^3) \\ \therefore y' &= 3x^2 e^{x^3} \end{aligned}$$

ตอบ

51. กำหนดให้ $y = 3^{-x^2}$ จงหาค่า y'

วิธีทำ $y = 3^{-x^2}$

$$\begin{aligned} y' &= \frac{dy}{dx} = \frac{d}{dx} (3^{-x^2}) \\ &= 3^{-x^2} \ln 3 \frac{d}{dx} (-x^2) \\ \therefore y' &= -2x (\ln 3) 3^{-x^2} \end{aligned}$$

52. กำหนดให้ $y = \ln 3x^5$ จงหา y'

วิธีทำ $y = \ln 3x^5$

$$\begin{aligned} y' &= \frac{d}{dx} (\ln 3x^5) \\ &= \frac{1}{3x^5} \cdot \frac{d}{dx} (3x^5) \\ &= \frac{1}{3x^5} 3 \frac{d}{dx} (x^5) \\ \therefore y' &= \frac{5}{x} \end{aligned}$$

53. กำหนดให้ $y = \ln(\ln \tan x)$ จงหา y'

วิธีทำ $y = \ln(\ln \tan x)$

$$\begin{aligned} y' &= \frac{d}{dx} [\ln(\ln \tan x)] \\ &= \frac{1}{\ln(\tan x)} \cdot \frac{d}{dx} (\ln \tan x) \\ &= \frac{1}{\ln(\tan x)} \cdot \frac{1}{\tan x} \frac{d}{dx} (\tan x) \\ &= \frac{1}{\tan x \cdot \ln(\tan x)} \cdot \sec^2 x \frac{dx}{dx} \\ &= \frac{\cos x}{\sin x \cdot \ln(\tan x)} \cdot \frac{1}{\cos^2 x} \\ &= \frac{1}{\sin x \cos x \ln(\tan x)} \\ &= \frac{1}{\frac{1}{2} \sin 2x \ln(\tan x)} \quad \because \sin A \cos A = \frac{1}{2} \sin 2A \\ \therefore y' &= \frac{2}{\sin 2x \ln(\tan x)} \end{aligned}$$

ตอบ

54. กำหนดให้ $y = x \ln x - x$ จงหา y'

วิธีทำ $y = x \ln x - x$

$$\begin{aligned} y' &= \frac{d}{dx} (x \ln x) - \frac{dx}{dx} \\ &= x \frac{d}{dx} (\ln x) + \ln x \frac{dx}{dx} - 1 \\ &= x \frac{1}{x} \frac{dx}{dx} + \ln x - 1 \\ \therefore y' &= 1 + \ln x - 1 = \ln x \end{aligned}$$

ตอบ

55. กำหนดให้ $f(x) = (\log x)^3$ จงหา $f'(x)$

วิธีทำ $f(x) = (\log x)^3$

$$\begin{aligned} f'(x) &= \frac{d}{dx} (\log x)^3 \\ &= 3(\log x)^2 \frac{d}{dx} (\log x) \\ &= 3(\log x)^2 \cdot \frac{1}{x} \cdot \log e \frac{dx}{dx} \end{aligned}$$

$$\therefore f'(x) = \frac{3}{x} (\log e)(\log x)^2$$

ตอบ

56. กำหนดให้ $y = e^{-x} \cos x$ จงหา $\frac{dy}{dx}$

วิธีทำ $y = e^{-x} \cos x$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [e^{-x} \cos x] = e^{-x} \frac{d}{dx} (\cos x) + \cos x \frac{d}{dx} (e^{-x}) \\ &= e^{-x} (-\sin x) \frac{dx}{dx} + \cos x e^{-x} \frac{d}{dx} (-x) \\ &= -e^{-x} \sin x - \cos x e^{-x} \\ \therefore y' &= -e^{-x} (\sin x + \cos x)\end{aligned}$$

ตอบ

57. กำหนดให้ $y = \arcsin e^x$ จงหา y'

วิธีทำ $y = \arcsin e^x$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [\arcsin e^x] \\ &= \frac{1}{\sqrt{1-(e^x)^2}} \frac{d}{dx} (e^x) \\ \therefore \frac{dy}{dx} &= \frac{e^x}{\sqrt{1-e^{2x}}}\end{aligned}$$

ตอบ

58. กำหนดให้ $y = e^{\sin 3x}$ จงหาค่า y'

วิธีทำ $y = e^{\sin 3x}$

$$\begin{aligned}y' &= \frac{d}{dx} [e^{\sin 3x}] \\ &= e^{\sin 3x} \frac{d}{dx} (\sin 3x) \\ &= e^{\sin 3x} \cos 3x \frac{d}{dx} (3x) \\ \therefore y' &= 3 \cos 3x e^{\sin 3x}\end{aligned}$$

ตอบ

59. กำหนดให้ $f(x) = \log(\log x)$ จงหา $f'(x)$

วิธีทำ $f(x) = \log(\log x)$

$$\begin{aligned}f'(x) &= \frac{d}{dx} \log(\log x) \\ &= \frac{1}{\log x} \cdot \log e \frac{d}{dx} (\log x) \\ &= \frac{1}{\log x} \cdot \log e \cdot \frac{1}{x} \log e \frac{dx}{dx} \\ \therefore f'(x) &= \frac{(\log e)^2}{x \log x}\end{aligned}$$

ตอบ

60. กำหนดให้ $y = \cos \sin x$ จงหา y'

วิธีทำ $y = \cos \sin x$

$$\begin{aligned}y' &= \frac{d}{dx} \cos \sin x = -\sin \sin x \frac{d}{dx} \sin x \\ &= -\sin \sin x \cos x\end{aligned}$$

ตอบ